Sets

Set theory includes a formal language for writing expressions about sets.

The objects of the language are sets and the combinators (operators) include intersection, union, set difference and complement.

Every (mathematical and non-mathematical) language has grammatical rules dictating how one can create valid expressions in the language.

What are the rules for sets?

Set Theory: Objects

A set is a collection of distinct related items.

A set is presented within curly braces and each item is delimited by a comma as in {1, 3, 2}. The order of the items is unimportant.

Thus {1, 4} = {4, 1}.

Example Sets:

The set of integers.

The set of students.

The set of countries with a palindromic name.

The set of operating systems.

The set of all sets.

Set Theory: Objects

A set is either empty or non-empty.

Empty Set Notation

{ } or ϕ

Some Non-empty Sets

{1, 2, 3}	The set containing 1, 2 and 3.
{BABS, BAAF, BIT, BABM}	The set of acronyms of UG courses.

Set Theory: Operators

Before discussing operators we need some accepted terminology. We refer to an item of a set as an element of the set and write:

$$x \in S$$

which is read x is an element of the set S.

The converse (not an element) is written:

$$x \notin S$$

Set Theory: Operators

Now let S, T and W be arbitrary sets.

Intersection

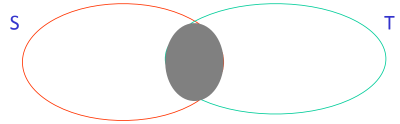
If one wants to know what elements are common to both sets then one wants the intersection of the sets written:

$$S \cap T$$

Set Theory: Pictures of Operator Application

Intersection

If one wants to know what elements are common to both sets then one wants the intersection of the sets drawn:



The above is known as a Venn diagram.

Set Theory: Operators

Now let S, T and W be arbitrary sets.

Union

If one wants to know what elements are in one or both sets then one wants the union of the sets written:

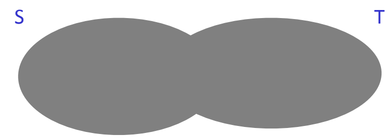
$$S \cup T$$

Set Theory: Pictures of Operator Application

Union

If one wants to know what elements are in one or both sets then one wants the union of the sets

drawn:



Set Theory: Operators

Let Un represent the universal set (the set which contains all the elements that other sets can contain in the given problem)

Complement

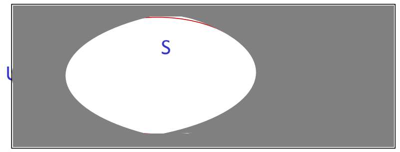
If one wants to know what elements are not in a set S but are in the universal set Un then one writes:

$$S^c$$

Set Theory: Pictures of Operator Application

Complement

If one wants to know what elements are not in a set S but are in the universal set Un then one draws:



Set Theory: Operators

Now let S, T and W be arbitrary sets.

Set Difference

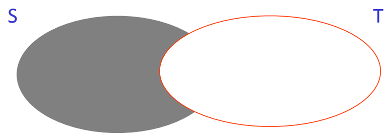
If one wants to know what elements are in one set but not another then one wants the set difference of the sets written:

$$S - T \quad \text{or} \quad S \setminus T$$

Set Theory: Pictures of Operator Application

Set Difference

If one wants to know what elements are in one set but not another then one wants the set difference of the sets drawn:



Set Theory: Subsets

Let S and T be arbitrary sets.

Subset

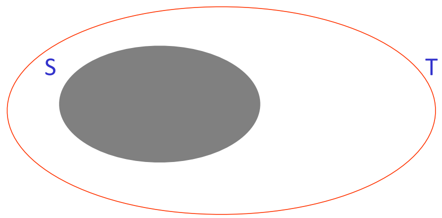
If the set S only contains elements of the set T (or equivalently doesn't contain any elements not in the set T) we say that S is a subset of T and write:

$$S \subseteq T$$

Set Theory: Picture of a Subset

Subset

If the set S only contains elements of the set T (or equivalently doesn't contain any elements not in the set T) we say that S is a subset of T and draw:



Set Theory: Subsets

Note:

One can create a subset of a set W by removing zero or more elements from W.

That is, a set is always a subset of itself, and the empty set is a subset of every set.